CERN-TH.7497/94
arch-ive/9411332

Semi-analytical and Monte Carlo results for the production of four fermions in e^+e^- collisions

Guido MONTAGNA[a], Oreste NICROSINI[b,1], Giampiero PASSARINO[c] and Fulvio PICCININI[d]

[a] INFN, Sezione di Pavia, Italy
[b] CERN, TH Division, Geneva, Switzerland
[c] Dipartimento di Fisica Teorica, Università di Torino and INFN, Sezione di Torino, Turin, Italy
[d] Dipartimento di Fisica Nucleare e Teorica, Università di Pavia, and INFN, Sezione di Pavia, Italy

Abstract

In view of the forthcoming experiments at high energy e^+e^- colliders, LEP2 and beyond, the process $e^+e^- \to W^+W^- \to 4f$ has been investigated for *off-shell* W's. After deriving the matrix element for the process within the helicity amplitude formalism, QED radiative corrections to the initial state leptons have been included at the leading logarithmic level in the framework of the electron structure functions. Cross sections and distributions are computed by means of both a semi-analytical and a Monte Carlo code, allowing for several cuts of experimental interest. A Monte Carlo event generator has also been built, for simulation purposes. Some illustrative numerical results are included and shortly described.

Submitted to Physics Letters B

CERN-TH.7497/94
November 1994

[1]On leave from INFN, Sezione di Pavia, Italy.

In the context of the experiments planned at e^+e^- colliders such as LEP2 and beyond, the process $e^+e^- \to W^+W^- \to 4\,f$ plays a special role since it will be used for an accurate measurement of the W-mass and hopefully to test the non-Abelian trilinear couplings of the standard model as well as the effects of non-standard "anomalous" couplings between the gauge bosons. In order to achieve these experimental goals, accurate predictions for the observables of the reaction $e^+e^- \to W^+W^- \to 4\,f$ are mandatory and in particular the contribution of all radiative corrections should be kept well under control.

For the above reasons a number of studies devoted to four fermions production in electron-positron collisions, including the main effects of radiative corrections, have recently appeared in the literature and they can be roughly grouped in two main lines, which are in some sense complementary:

- semi-analytical approaches [1, 2],
- Monte Carlo approaches [3, 4, 5].

Previous calculations of the higher order QED corrections to $e^+e^- \to W^+W^-$, including the influence of the finite W width, were performed in [6, 7]. The complete study of the reaction $e^+e^- \to 4\,$fermions clearly represents a multi-step program and the aim of this letter is to summarize our novel theoretical approach to it, which includes the description of $e^+e^- \to W^+W^- \to 4\,$fermions with initial state electromagnetic radiation in the leading log approximation. A complete analysis of the *background* processes and of the *exact* $\mathcal{O}(\alpha)$ radiative corrections are currently under investigation. Our main goal in this letter will be to present an illustrative sample of the numerical results obtained so far by means of a semi-analytical and a Monte Carlo program. We start with a brief description of the basic ingredients.

We have undertaken a general study of the process $e^+e^- \to 4$ fermions with the aim of producing and discussing a wide number of realistic distributions. As a first step we briefly introduce some basic concepts and apply them to the *double-resonating* diagrams $e^+e^- \to W^+W^- \to 4\,$fermions. The main emphasis in our approach is on a covariant description of the kinematics, which allows us to translate any sort of kinematical cut directly into an analytical constraint on the integration variables. As for any $2 \to 4$ process we can introduce 15 invariants $s_{ij} = -(\epsilon_i\, p_i + \epsilon_j\, p_j)^2$ where $p_i, i = 1, \ldots, 6$

are all the momenta assumed to flow inwards and $\epsilon_i = \pm 1$ for incoming (outgoing) particles. There are 6 linear and 1 non-linear constraints among the s_{ij} with a total of 8 independent invariants, one of which is of course the invariant mass of the event, s. Using standard techniques all the components of the momenta (up to a sign ambiguity) can be expressed in terms of the s_{ij} and thus in terms of the independent invariants. This is why we can express any kinematical cut as an analytical constraint on the integration variables and why our multi-dimensional phase space can be treated exactly.

For $e^+(p_1)e^-(p_2) \to f_1(p_3) + \dots f_4(p_6)$ the independent variables used to describe the process are, besides $s = -(p_1 + p_2)^2$, the following ones: $z_1 = x_{13} + x_{23}$, $z_3 = x_{15} + x_{25}$, $\mu_1^2 = x_{34}$, $\mu_2^2 = x_{56}$, $\mu_3^2 = x_{35}$, $\tau_1 = x_{13}$ and $\tau_2 = x_{14}$ where we have defined $x_{ij}s = s_{ij}$. In the c.m. system they have a rather simple interpretation, namely

$$z_i = \frac{2E_i}{\sqrt{s}}, \qquad i = 1, 3, \tag{1}$$

where E_i are the energies of particles 1 and 3, which correspond to the *down* fermion coming from the decay of W^- and to the *up* fermion coming from the decay of W^+, respectively;

$$\mu_i^2 = \frac{s_i}{s}, \qquad i = 1, 2, \tag{2}$$

where s_1 and s_2 are the invariant masses of W^+ and W^- respectively;

$$\tau_i = \frac{E_i}{\sqrt{s}}(1 - \cos\vartheta_i^+), \qquad i = 1, 2, \tag{3}$$

where ϑ_i^+ is the scattering angle of the i-th particle with respect to the incoming positron beam;

$$\mu_3^2 = \frac{s_{13}}{s}, \tag{4}$$

where s_{13} is the invariant mass between the particles 1 and 3. Given the independent variables, the further choice of the solution of the non-linear constraint allows the reconstruction of the kinematics in the laboratory frame. The explicit solution of the kinematics allows us to translate several cuts of experimental interest in terms of boundaries of the independent variables phase space; such a translation is of great importance from the point of view of numerical integration when adopting a semi-analytical approach.

When including initial state radiation, two more independent variables $x_{1,2}$, which represent the energy fraction carried by the incoming electron and positron after the radiation, have to be added to the previous set. In this case the above independent variables are referred to the c.m. system of the hard scattering process and in this frame the dependent variables are derived by explicit solution of the kinematics. By means of a Lorentz boost the whole kinematics in the laboratory system can finally be reconstructed thus allowing the boundaries of the phase space in the presence of realistic cuts to be defined also for the radiative process.

Our matrix element contains for the moment only the three *double-resonant* diagrams in Born approximation, but there is no limitation of the method that will prevent us from a straightforward generalization to all the *background* processes. The strategy adopted is to use one of the many formulations of the so-called helicity method [8], which has the advantage of working directly with invariants. Roughly speaking, each diagram can be written as

$$d_i(\lambda, \rho_f, \rho'_f) = (2\pi)^4\, ig^4 \eta_i \kappa_i \Delta_i \bar{v}_{-\lambda} \Gamma_{in} u_\lambda \bar{u}_{\rho_f} \Gamma_f v_{-\rho_f} \bar{u}_{\rho_{f'}} \Gamma_{f'} v_{-\rho_{f'}} \tag{5}$$

where λ, ρ_f and $\rho_{f'}$ are helicity labels, η_i is the sign of the diagram, κ_i stands for a collection of constants and Δ_i for the collection of propagators. Finally Γ_{in}, Γ_f and $\Gamma_{f'}$ are strings of γ matrices. Once each diagram is rewritten as a trace, we explicitly insert for the bilinears the corresponding expression (valid up to an overall phase), as for instance

$$v_\lambda(p)\bar{u}_\lambda(q) = -\frac{1}{2}\frac{1}{(2p\,q)^{1/2}}\left(1 - \lambda\,\gamma^5\right) \not{p}\not{q} \tag{6}$$

and the trace operation will convert everything in terms of invariants, which in turn are related to the integration variables. According to the above procedure the three diagrams of the process $e^+e^- \to W^+W^- \to 4\,f$ have been computed by means of `SCHOONSCHIP` [9] and FORTRAN-coded to get the squared matrix element numerically . The effect of the electromagnetic initial state radiation is treated in the language of the QED structure functions [10, 11], where the corrected cross section can be written in the following form:

$$\sigma(s) = \int dx_1\, dx_2\, D(x_1, s) D(x_2, s) d[PS] \frac{d\sigma}{d[PS]}. \tag{7}$$

In the latter equation $d[PS]$ denotes the volume element in the 7-dimensional phase space connected to the independent variables of the hard scattering reaction in the c.m. frame; $D(x, Q^2)$ provides the probability of finding, inside a parent electron, an electron of energy fraction x with virtualness Q^2. The explicit expression which we adopt to treat initial state radiation is given by

$$\begin{aligned} D(x,s) &= \frac{\exp\left\{\frac{1}{2}\beta\left(\frac{3}{4}-\gamma_E\right)\right\}}{\Gamma\left(1+\frac{1}{2}\beta\right)} \frac{\beta}{2}(1-x)^{\frac{\beta}{2}-1} - \frac{\eta}{4}(1+x) \qquad (8) \\ &+ \frac{1}{32}\eta^2 \left[-4(1+x)\ln(1-x) + 3(1+x)\ln x - 4\frac{\ln x}{1-x} - 5 - x\right], \end{aligned}$$

with

$$\beta = 2\,\frac{\alpha}{\pi}\,(L-1), \qquad \eta = 2\,\frac{\alpha}{\pi}\,L, \tag{9}$$

where $L = \ln(s/m^2)$ is the collinear logarithm, γ_E is the Euler constant and $\Gamma(z)$ is the gamma function. The first exponentiated term of Gribov–Lipatov form describes multiphoton soft radiation, the second and third ones hard collinear bremsstrahlung. In the structure function the arbitrary scale Q^2 is set equal to s and the hard photon terms are taken with the coefficient η instead of β in order to treat the hard radiation in leading log approximation (in this approximation, problems connected with gauge invariance are avoided).

As a consequence of the high dimensionality of the integration, a Monte Carlo strategy could be in principle competitive with a deterministic numerical procedure, as far as the balance between number of calls and accuracy are concerned. Both of the approaches have been adopted to cross-check and put under control the numerical results. The Monte Carlo integration (based on the random numbers generator `RANLUX` [12]) has been performed according to the importance sampling technique in order to cure the peaking behaviour of the integrand. This strategy has been helpful also for the semi-analytical approach, which has been carried out by means of the NAG [13] routine `D01GDF`. As a result, the deterministic procedure turns out to be more efficient with regard to a totally inclusive set-up and in the presence of cuts on the final state invariant masses and/or energies and/or scattering angles of the outgoing fermions. On the other hand, the Monte Carlo allows for a complete and more flexible control of any kind of experimental cut. Furthermore, the Monte Carlo routine can also be used as an event generator for simulation purposes. In this case the events, defined as the components of the four final state particles momenta, plus the radiative variables x_1 and x_2, plus $\sqrt{s}$, are stored into proper n-tuples. A detailed description of the formulation and of the technical solutions adopted will be presented elsewhere.

In the following we show some illustrative results obtained by means of the two numerical procedures described above. Our set of input parameters is $M_Z = 91.1887\,\mathrm{GeV}$, $M_W = 80.22\,\mathrm{GeV}$, $\Gamma_Z = 2.4974\,\mathrm{GeV}$, $\Gamma_W = 2.08\,\mathrm{GeV}$ [14].

In Fig. 1 the total cross section, for W decaying in all possible final states (excluding top production), in Born approximation (dashed line) is compared with the initial state QED-corrected one (solid line) as a function of the c.m. energy for a fully extrapolated set-up. Concerning the tree level prediction, the dashed line result has been obtained by a two-fold integration of the formula given in ref. [15] while the solid line derives from a nine-fold integration of the $e^+e^- \to W^+W^- \to 4f$ matrix element performed by our semi-analytical code. The Monte Carlo results are represented by the open circles, showing a fully satisfactory agreement in both the Born and QED-corrected case. For such a configuration, the effect of initial state radiation is to reduce the peak cross section by about 10%, to slightly shift the peak position towards higher energies and to determine the appearance of a radiative tail as a typical convolution effect. For the QED-corrected total cross section we compared our results, obtained by properly adapted structure functions, with the values quoted in [1] and found very good agreement.

Figure 2 shows the W^- invariant mass distribution $d\sigma/dM_-$ ($M_-^2 = -(q_1+q_2)^2$) at the Born level for two c.m. energies in the LEP2 regime, namely $\sqrt{s} = 175$ GeV (dotted line) and $\sqrt{s} = 190$ GeV (dashed line). In order to single out the contribution of the initial state radiation, the corrected invariant mass distribution at $\sqrt{s} = 175$ GeV (solid line) is also plotted for comparison. As in Fig. 1, the Monte Carlo predictions given by the markers fully agree with the semi-analytical results used to produce the continuous lines.

The effect of some realistic experimental cuts is shown for the total cross section (Fig. 3) and the invariant mass distribution at $\sqrt{s} = 175$ GeV (Fig. 4) of $e^+e^- \to W^+W^- \to 4f$ where both W's are required to decay hadronically. All the curves include the effect of the initial state leading log QED corrections. The experimental conditions considered are the following: no cut (solid line), cuts on the energies of all the four final state fermions $E^i_{min} = 25$ GeV (dashed line), constraint on the invariant masses of both W's $77 \le M_\pm \le 83$ GeV for the total cross section and on the W^+ invariant mass $77 \le M_+ \le 83$ GeV for the W^- invariant mass distribution (close-dotted line), acceptance cuts on the scattering angles of the produced fermions $40^\circ \le \vartheta_i \le 140^\circ$ (wide-dotted line). Also for such realistic configurations the agreement between the semi-analytical (lines) and Monte Carlo (markers) integrations is fully satisfactory pointing out the flexibility of our semi-analytical code.

In Figs. 5–9 we show some distributions of experimental interest for LEP2 physics, which have been obtained processing the n-tuple created by the event generator at $\sqrt{s} = 175$ GeV taking into account the emission of initial state radiation. They refer to a sample of 10^5 four fermion events and show the effects on the considered distributions of three different selection criteria: fully extrapolated set-up (white histogram), cuts on the scattering angles of $1,2$ particles coming from the decay of the W^- boson ($40^\circ \le \vartheta_{1,2} \le 140^\circ$) (dark histogram) and cut on the invariant mass of the W^- boson ($77 \le M_- \le 83$ GeV) (grey histogram). The effect of the cuts is shown for the W^- angular distribution (Fig. 5), the energy distribution of the fermion from W^- decay (Fig. 6), the distribution of the relative angle of the decay products of the W^- boson (Fig. 7), the scattering angle of the fermion produced by W^- decay (Fig. 8) and, finally, the photon energy distribution (Fig. 9).

Acknowledgements – We would like to thank the INFN, Sezioni di Pavia and Turin, for having provided computer resources. We are also grateful to Giorgio Fumagalli and Valerio Vercesi for helpful assistance in numerical and graphical problems.

References

[1] D. Bardin, M. Bilenky, A. Olchevski and T. Riemann, Phys. Lett. B308 (1993) 403; CERN preprint CERN-TH.7295/94 and DESY preprint DESY 94-093.

[2] W. Beenakker and A. Denner, DESY preprint DESY 94-051 and references therein.

[3] F. A. Berends, R. Kleiss and R. Pittau, Nucl. Phys. B424 (1994) 308 and B426 (1994) 344.

[4] H. Anlauf et al., Comput. Phys. Commun. 79 (1994) 487.

[5] G. J. van Oldenborgh, P. J. Franzini and A. Borrelli, Comput. Phys. Commun. 83 (1994) 14.

[6] M. Cacciari, A. Deandrea, G. Montagna and O. Nicrosini, Z. Phys. C52 (1991) 421.

[7] V. S. Fadin, V. A. Khoze and A. D. Martin, Phys. Lett. B311 (1993) 311.

[8] G. Passarino, Nucl. Phys. B237 (1984) 249.

[9] `SCHOONSCHIP`, a program for symbol handling by M. Veltman, see H. Strubbe, Comput. Phys. Commun. 8 (1974) 1.

[10] E. A. Kuraev and V. S. Fadin, Sov. J. Nucl. Phys. 41 (1985) 466;
G. Altarelli and G. Martinelli, in *Physics at LEP*, CERN Report 86-02, J. Ellis and R. Peccei eds. (CERN, Geneva 1986).

[11] O. Nicrosini and L. Trentadue, Phys. Lett. B196 (1987) 551; Z. Phys. C39 (1988) 479. For a review see also
O. Nicrosini and L. Trentadue, in *Radiative Corrections for e^+e^- Collisions*, J. H. Kühn ed. (Springer, Berlin, 1989), p. 25; in *QED Structure Functions*, G. Bonvicini ed., AIP Conf. Proc. No. 201 (AIP, New York, 1990), p. 12; O. Nicrosini, ibidem p. 73.

[12] F. James, Comput. Phys. Commun. 79 (1994) 111.

[13] NAG Fortran Library Manual Mark 15 (Numerical Algorithms Group, Oxford, 1991).

[14] Review of Particles Properties, Phys. Rev. D50 (1994) 1173.

[15] T. Muta, R. Najima and S. Wakaizumi, Mod. Phys. Lett. A1 (1986) 203.

Figure Captions

Figure 1. The total cross section of $e^+e^- \to W^+W^- \to 4f$ without (dashed line) and with (solid line) initial state leading log QED corrections as a function of the c.m. energy and for W's decaying into all possible final states (top excluded). The open circles are the results of our Monte Carlo program in comparison with the integration of the analytic formula of ref. [15] for the tree level approximation and with the predictions of our semi-analytical code for the radiative case.

Figure 2. The W^- invariant mass distribution $d\sigma/dM_-$ ($M_-^2 = -(q_1+q_2)^2$) at the Born level for two c.m. energies $\sqrt{s} = 175$ GeV (dotted line) and $\sqrt{s} = 190$ GeV (dashed line). The solid line is the QED-corrected invariant mass distribution at $\sqrt{s} = 175$ GeV. The Monte Carlo integration results are given by the markers.

Figure 3. The effect of some experimental cuts on the total cross section of $e^+e^- \to W^+W^- \to 4f$ including initial state leading log QED corrections as a function of the c.m. energy, and for both W bosons decaying hadronically. The Monte Carlo integration results are given by the markers.

Figure 4. The same as in Fig. 3 for the invariant mass distribution at $\sqrt{s} =$ 175 GeV.

Figure 5. The effect of specific selection criteria on the W^- angular distribution at $\sqrt{s} = 175$ GeV with QED corrections included: no cuts (white histogram), cuts on the scattering angles of $1,2$ particles coming from the decay of the W^- boson ($40^\circ \leq \vartheta_{1,2} \leq 140^\circ$) (dark histogram) and cut on the invariant mass of the W^- boson ($77 \leq M_- \leq 83$ GeV) (grey histogram).

Figure 6. The same as Fig. 5 for the fermion energy distribution.

Figure 7. The same as Fig. 5 for the relative angle of the decay products of the W^- boson.

Figure 8. The same as Fig. 5 for the fermion scattering angle.

Figure 9. The same as Fig. 5 for the photon energy distribution ($E_\gamma \leq$ 25 GeV).